AMYGDALA $z \mapsto z^2 + c$

A Newsletter of Fractals & $\mathcal{M}$ (the Mandelbrot Set)
Issue #27
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In This Issue...

Robert Hill's *Dithering As A Shading Technique* reveals in detail the techniques used to generate the images and painterly backgrounds seen in the four slides in set #26, and in slide A2422 from set #27, distributed with this issue.

Michael Freeman's *Mandelbrot And Julia Sets Of Elliptic Functions* discusses the fractals that result from iteration over the Elliptic functions: a refreshing change from the quadratic, polynomial and rational functions most often used for iteration.

In *How To Spin $\mathcal{M}$-sets On A Monitor Screen In Real Time,* John MacManus discusses a program for rotating 3-D representations of Mandelbrot set views.

Dithering As A Shading Technique For Fractal Images

Robert Hill

Although there has been little mention of dithering in the literature on the display of Mandelbrot and Julia sets, this technique has much potential for producing beautifully shaded, often 3-dimensional appearing versions of these objects. This term, *dithering,* originally denoting a state of trembling or agitation, has come to mean in the art world the introduction of noise or randomness into the coloring of various elements of a picture. This randomness is a means of obtaining greater color fidelity at the expense of spatial fidelity or image sharpness. In representational art this is desirable to show the gradations in brightness of unevenly lit or shaded surfaces, for example. In the case of abstract computed images such as Mandelbrot sets, where the color is arbitrary, dithering can create a similar effect of shading, thereby giving a physical, 3-dimensional appearance to the mathematical object. In so doing, dithering replaces the discrete color steps defined by the Mandelbrot algorithm with a smooth appearing, sloping surface.

There are many ways of dithering a picture of a Mandelbrot or Julia set. For any of them, however, the effect obtained is strongly dependent on the contrast in color and brightness of the adjacent colors involved in the dithering. Where the differences in value and hue are small, the result will be a very smooth appearing surface. If the differences are larger, the surface will take on a grainy or speckled character. Slide A2421 of slide set S25 affords a good example of this. The same dithering algorithm was applied to the entire picture, but the peripheral "background" gives an effect quite different from that of the central figure.

The very high contrast inherent in black and white pictures makes it impossible to demonstrate the more subtle effects seen with color. For this reason the reader is referred to the slide supplement for some examples. This importance of contrast also

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makes it evident that in selecting a palette for a picture, the effect desired from the dithering must be taken into consideration.

The simplest way to introduce dithering into a picture is merely to add a random term to the expression defining the color to be assigned to a pixel, as

```
color = colorNumber + rnd*k
```

For most purposes, *k* should be selected so that *color* often, but not always, assumes the next higher color number, or perhaps occasionally jumps to a still higher color. The greater the value of *k*, however, the greater the effect of obscuring the sharpness of the Mandelbrot image, so that values greater than about 2 are not satisfactory.

The Mandelbrot and Julia sets do have an advantage: outlying areas of low dwell are the regions where dithering is most effective. In the convoluted fractal regions where contour lines are very close together, the variability inherent in the set overwhelms the dithering, so that the sharp edges are still retained where sharpness is most wanted.

Conversely, in the outlying regions where contours are far apart, the contours tend to disrupt the dithering, and it is often desirable to obscure the contour lines as much as possible. One way to do this is to randomize the escape function, e.g.

$x^2 + y^2 < 4 + rnd \cdot k.$

This has the effect of spreading the contour into a band of width *k*. In combination with the dithering routine, this technique can completely obscure the outlying contour lines while having little effect on the central figure.

The Slides (S26, continued)

Bob Hill's four slides were sent out with Amy #26 without technical data. Since then Bob has sent along the following information. The names of the slides are mine, not his.

See also Bob's article, *Dithering as a shading technique for fractal images,* elsewhere in this issue.

A2412: *Fierce moth.* A detail of the Julia set located at $0.2525 + 0i$, for the function $z^2 + c$. The texture arises from dithering using a "checkerboard" formula of the form:

```
if px  mod 2 + py mod 2 = 1
then Color = D - 10*rnd
else Color = D + 10*rnd
```

where px, py = pixel coords, D = current dwell.

The number of iterations, D_{max}, was 400. Note that the dithering was applied only when $D < 235$.

A2417: *Oils on rough canvas.* A portion of the Julia set at $0.085 + 1.007i$ of the function $(z - \frac{z^3}{3!} + \frac{z^5}{5!}) \cdot c$, which is the power series for $\sin z$ truncated to three terms. The dithering formula was similar to that above, but was used in combination with a randomized escape limit function (ELF), namely $x^2 + y^2 < 5 + rnd \cdot 6$.

Note the texture resembling chalk on canvas; it is more conspicuous here, probably due to greater contrast between adjacent colors.

A2413: *Prickly pear.* The central region of the Julia set at $1 + 0.1i$ for the same function as A2417, except that the two terms in the x, y expression have different divisors: 60 rather than 24. Thus the function differs somewhat from $\sin z$. The dithering formula is also like that of A2417. A distinctive feature here is the ELF, with limit 16: it is the iterated function itself. This gives rise to the asymmetric swirls on either side of the central spiral.

A2421: *Spider's web.* This is a detail (relative magnification 3.3) of Bob's A2422. See below for a description of A2422.

The Slides (S27)

A903 (James Loyless): *San Marco.* The insides of Mandelbrot's Dragon of San Marco are mapped using the minimum value of z as the inside color / alternation criterion. Center =0 x 0.93, aspect ratio = 1.444, dwell limit = 250, z_{min} multiplier = 250.

A908 (James Loyless): *Z–eye.* An eye from Sea Horse land. This one is my wife's favorite. Center = –0.74355+0.131995 x 58,800, aspect ratio = 1.41, dwell limit = 1000.

A2422 (Bob Hill): *Dragon.* A Julia set of $\sin z$ (truncated) located at $1.13 + 0.495i$. The dithering is similar to A2413, but with an added $0.015 \cdot py$

term (where *py* is the pixel coordinate), giving rise to the color gradation from top to bottom. The ELF is randomized as well. This calculation differs, however, in that the function $z^2 + c$ was used for the first two iterations. This slide is the "big picture" of which A2421 is a detail.

A4708 (Gérardin): *PHILOSOPHER'S EMERALD.* Julia; c = -1.476 (FRACTINT + spherical projection).

Mandelbrot And Julia Sets Of Elliptic Functions

Michael Freeman

After seeing the beauty and complexity of Julia and Mandelbrot sets for the function $z \leftarrow z^2 + c$ it is natural to wonder what we would see after iterating other functions in a similar manner. Readers of Amygdala will be familiar with the results for other elementary functions, e.g. higher order polynomials, exponentials, and the circular functions $\sin z$ and $\cos z$. Recently I have been exploring Julia sets of elliptic functions. Although these are less familiar than the functions mentioned above, they have some very nice properties and are not too difficult to calculate.

The theory of elliptic functions, developed in the 19th century by Gauss, Jacobi, Weierstrass and other great mathematicians, is a wonderful accomplishment of mathematical analysis [Abr64], [Whi27]. For the reader familiar with complex numbers and elementary calculus I will attempt here to scratch the surface. (I apologize for any offence to serious mathematicians.)

Examples of periodic functions are familiar to most of us: the circular functions *sin* and *cos* with the period 2π (radians). Specifically, this means that $\sin z = \sin(z + 2\pi)$ for any complex number z. Less familiar at first glance are the hyperbolic functions *sinh* and *cosh*. These are defined in terms of exponentials: $\sinh z = \frac{e^z - e^{-z}}{2}$, $\cosh z = \frac{e^z + e^{-z}}{2}$. They are also periodic, but their period is along the imaginary axis, $2\pi i$. Because the properties of the exponential function, we have $\sinh(ix) = -i\sin x$, and $\cosh(ix) = \cos x$. The circular and hyperbolic functions are both periodic, each with a single period.

Elliptic functions are *doubly* periodic. That is, they have two distinct periods in different directions in the complex plane. In fact, the general definition of an elliptic function is: any function of a complex variable which is doubly periodic and which has poles, but no essential singularities, in the finite plane. (The terms "pole" and "essential singularity" refer to points where the value of the function blows up or goes to infinity. The distinction is that the limit of the function as we approach a pole behaves like an inverse power of z. Near an essential singularity, the behaviour is more complicated.) Another definition, apparently completely different, is in terms of the inverse of certain indefinite integrals. One way to define the circular and hyperbolic functions is as the inverses of integrals of the square root of quadratic functions, e.g.

$$u = \int_0^{\varphi} \frac{dt}{\sqrt{1 - t^2}}; \quad \varphi = \sin u.$$

This is generalized to elliptic functions by considering integrals of the square root of cubic or quartic functions.

I've investigated a type of elliptic function called *Jacobian.* These functions are useful; they represent the arc length of an ellipse and the displacement of a simple pendulum, among other things. The three commonest functions *sn, cn, dn* are defined as follows. Given $u = \int_0^{\varphi} \frac{dt}{\sqrt{1 - m\sin^2 t}}$, we define $\mathrm{sn}(u) = \sin\varphi$, $\mathrm{cn}(u) = \cos\varphi$, and $\mathrm{dn}(u) = \sqrt{1 - m\sin^2\varphi}$.

The parameter m ranges from 0 to 1. To show the dependence on m we write $\mathrm{sn}(u|m)$, $\mathrm{cn}(u|m)$, $\mathrm{dn}(u|m)$. When $m = 0$, the elliptic functions reduce to circular functions. When $m = 1$, the elliptic functions reduce to hyperbolic functions:

$$\begin{aligned} \mathrm{sn}(u\,|\,0) &= \sin u & \qquad \mathrm{sn}(u\,|\,0) &= \tanh u \\ \mathrm{cn}(u\,|\,0) &= \cos u & \qquad \mathrm{cn}(u\,|\,0) &= \mathrm{sech}\, u \\ \mathrm{dn}(u\,|\,0) &= 1 & \qquad \mathrm{dn}(u\,|\,1(0)\,) &= \mathrm{sech}\,\iota \end{aligned}$$

When m is between 0 and 1, these functions have periods along both real and imaginary axes. The periods depend on m: as m goes to 0, the real period goes to 2π and the imaginary period goes to $i\infty$. As m goes to 1, the real period becomes ∞ and the imaginary period becomes $2\pi i$. The periods can be found in terms of the complete elliptic integral, $u(\varphi)$, when $\varphi = \pi/2$. This quantity is denoted K and depends on m. Also, $K(1-m)$ is denoted K'. The real period is $4K$ and the imaginary period is $2iK'$, so

$$\begin{aligned} \mathrm{sn}(z) &= \mathrm{sn}(z+4K) = \mathrm{sn}(z+8K) = \ldots \\ &= \mathrm{sn}(z+2iK') = \mathrm{sn}(z+4iK') = \ldots \end{aligned}$$

In addition, there are poles at $z = iK'$ and at $2K + iK'$.

These functions have many interesting properties, similar to the circular functions. For example,

$$\mathrm{sn}(-u) = -\mathrm{sn}(u),\ \mathrm{cn}(-u) = \mathrm{cn}(u)$$

$$\frac{d}{du}\mathrm{sn}(u) = \mathrm{cn}(u)\cdot\mathrm{dn}(u)$$

$$\mathrm{sn}\,(iu\,|\,0) = i\frac{\mathrm{sn}(u\,|\,1-m)}{\mathrm{cn}(u\,|\,1-m)}$$

$$\mathrm{sn}(u+v) = \frac{\mathrm{sn}(u)\cdot\mathrm{cn}(v)\cdot\mathrm{dn}(v)+\mathrm{sn}(v)\cdot\mathrm{cn}(u)\cdot\mathrm{dn}(u)}{1-m\cdot\mathrm{sn}^2(u)\cdot\mathrm{sn}^2(v)}$$

The first four of these properties follow in a straightforward way from the definition. The fifth is less obvious; see Whittaker and Watson [Whi27] for the proof.

Calculating the elliptic functions

I used the method of the arithmetic-geometric mean (*agm* — see appendix) which involves calculating trig, inverse trig, and exponential functions. Naturally, this takes more time than $z^2 + c$; for speed I programmed my 83D87 math processor in assembler which led to acceptable calculation times (25 MHz) using the Mariani algorithm.

The actual functions iterated were: $c\cdot\mathrm{sn}(z\,|\,m)$, $c\cdot\mathrm{cn}(z\,|\,m)$, and $c\cdot\mathrm{dn}(z\,|\,m)$, where c is a complex parameter. Since these functions are doubly periodic in z, the Julia picture is doubly periodic in z as well — good for wallpaper designs. The appearance is quite sensitive to the choice of escape radius (and of course, c and m). I made sure *not* to check for escape before the first iteration.

Figure 1 shows the Julia set of $2.9\cdot\mathrm{cn}(z\,|\,0.2)$. The double periodicity can easily be seen, as well as the location of the poles. A maximum dwell of 100 was used. The escape radius is $\sqrt{20}$, as it is for the other figures as well.

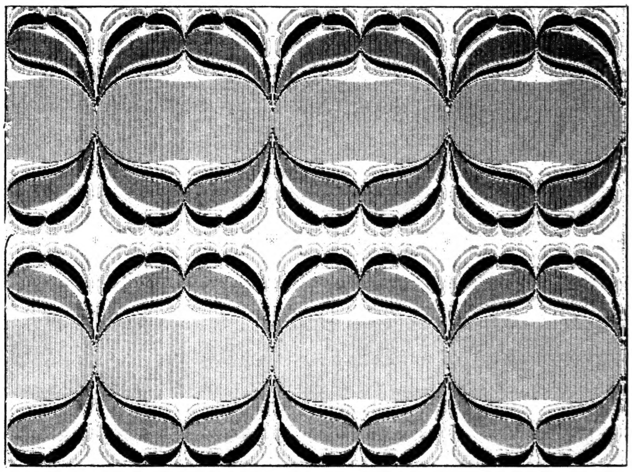


Figure 1. **The Julia Set of** $2.9\cdot\mathrm{cn}(z\,|\,0.2)$

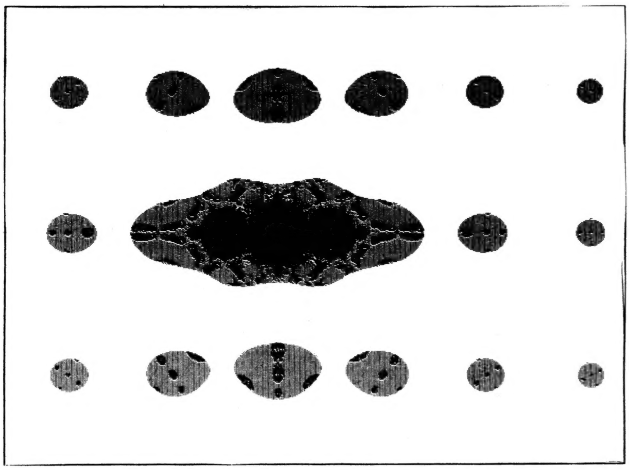


Figure 2. **Mandelbrot Set of** $c\cdot\mathrm{sn}(z\,|\,0.2)$

For Mandelbrot pictures, c is the screen point; but one change is made from the usual procedure: z cannot be set to 0 for the *sn* functions, because $\mathrm{sn}(0) = 0$. Instead, z is initialized to c. This works for $z^2 + c$ also, with one extra iteration. Figure 2 is the Mandelbrot set for $c \cdot \mathrm{sn}(z|0.2)$ between –13.3 and +13.3 on the real axis, and between –10 and 10 on the imaginary axis. As that figure shows, there is a Mandelbrot-like entity in the centre, and the double periodicity is not exact.

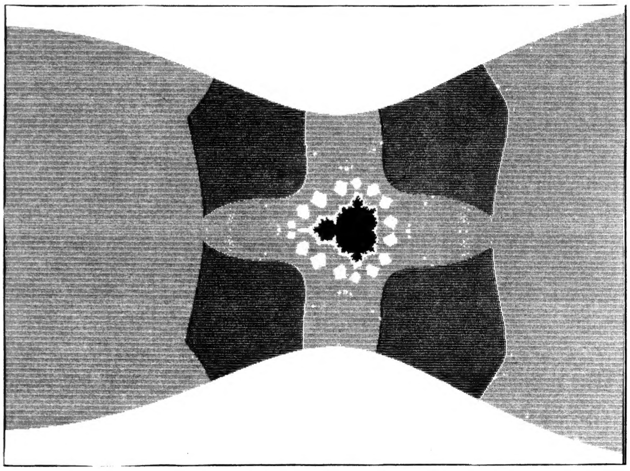


Figure 3. **Detail of Figure 2**

The dwell contours are very different from the usual sets. The window appearing in Figure 2 is blown up in Figure 3. Note the midget Mandelbrot. (For figures 2, 3, 4 the maximum dwell used was 50.).

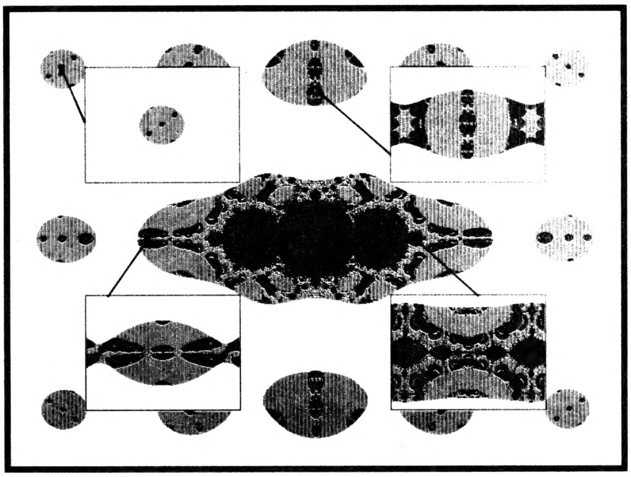


Figure 4. **Central Feature of Figure 2, with Julia Sets**

A relationship between points on the Mandelbrot picture and the Julia sets exists. Figure 4 shows a blowup of the central feature of Figure 2 with four Julia sets (these are calculated between -2 and +2 on the real axis, -1.5 and +1.5 on the imaginary axis.

For the elliptic functions, neither the Mandelbrot set nor the Julia sets from interior points are connected. Nonetheless, it seems that there might be a property that would play the role here that connectedness plays for $z^2 + c$.

One could investigate these functions more deeply; so far I have just begun.

Appendix: calculating *sn, cn, dn*

When the parameter m is less than 0.05, we use:

$$\mathrm{sn}(u|m) \cong \sin(u) - 0.25m(u - \sin(u)\cos(u))\cos(u)$$

$$\mathrm{cn}(u|m) \cong \cos(u) + 0.25m(u - \sin(u)\cos(u))\sin(u)$$

$$\mathrm{dn}(u|m) \cong 1 - 0.5m\sin^2 u$$

When the parameter m is greater than 0.95, we use:

$$\mathrm{sn}(u|m) \cong \tanh(u) + 0.25(1-m)(\sinh(u)\cosh(u) - u)\,\mathrm{sech}^2(u)$$

$$\mathrm{cn}(u|m) \cong \mathrm{sech}(u) - 0.25(1-m)(\sinh(u)\cosh(u) - u)\tanh(u)\,\mathrm{sech}(u)$$

$$\mathrm{dn}(u|m) \cong \mathrm{sech}(u) + 0.25(1-m)(\sinh(u)\cosh(u) + u)\tanh(u)\,\mathrm{sech}(u)$$

Otherwise, we use the agm method:

Given m, we create a table of triples (a_n, b_n, c_n) for $n = 0, 1, 2, \ldots, N$. We start with $a_0 = 1$, $b_0 = \sqrt{1-m}$, $c_0 = \sqrt{m}$, and iterate:

$$a_n = \frac{a_{n-1} + b_{n-1}}{2}$$

$$b_n = \sqrt{a_{n-1} b_{n-1}}$$

$$c_n = \frac{a_{n-1} - b_{n-1}}{2}$$

until c_N is less than 0.001. (N will not exceed 12.)

We then create a table of φ_n, starting with $\varphi_N = 2^N \cdot a_N \cdot u$, and computing the rest from the relation $\sin(2\varphi_{n-1} - \varphi_n) = \frac{c_n}{a_n} \sin(\varphi_n)$.

Finally, we have

$$\mathrm{sn}(u|\, m) = \sin(\varphi_0)$$

$$\mathrm{cn}(u|\, m) = \cos(\varphi_0)$$

$$\mathrm{dn}(u|\, m) = \frac{\cos(\varphi_0)}{\cos(\varphi_1 - \varphi_0)}$$

Now, we require the relationship between the functions of a complex z and those of its real and imaginary parts (x, y). These are:

$$\mathrm{sn}(x + iy) = \frac{s \cdot d_1 + icd \cdot s_1 \cdot c_1}{D}$$

$$\mathrm{cn}(x + iy) = \frac{c \cdot c_1 - isd \cdot s_1 \cdot d_1}{D}$$

$$\mathrm{dn}(x + iy) = \frac{d \cdot c_1 \cdot d_1 - imsc \cdot s_1}{D}$$

where $D = c_1^2 + ms^2 s_1^2$, $s = \mathrm{sn}(x|\, m)$, $c = \mathrm{cn}(x|\, m)$, $d = \mathrm{dn}(x|\, m)$, $s_1 = \mathrm{sn}(y|\, 1 - m)$, $c_1 = \mathrm{cn}(y|\, 1 - m)$, and $d_1 = \mathrm{dn}(y|\, 1 - m)$.

References

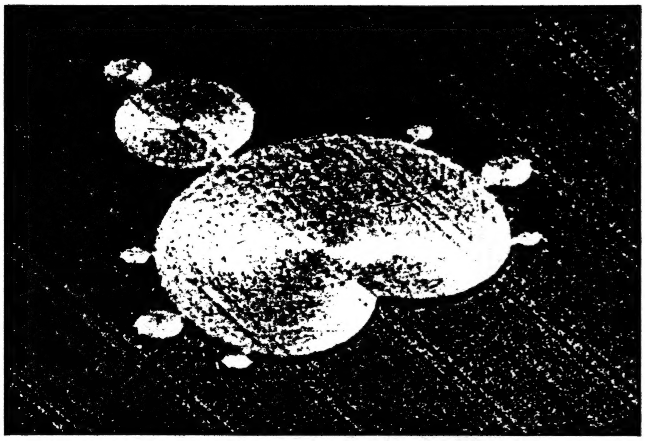
[Abr64] *Handbook of Mathematical Functions*; edited by Abramowitz and Stegun; National Bureau of Standards, 1964. Chapters 16,17.

[Whi27] E. T. Whittaker & G. N. Watson, *A Course of Modern Analysis*; Cambridge, 1927. Chapter XXII.

[Wol88] S. Wolfram, *Mathematica*; Addison Wesley, 1988.

Corny $\mathcal{M}$

The world's most mathematical hoaxers struck in the cornfields of Cambridge last week. Rather than a mere corn circle, they created a corn Mandelbrot set — the most complicated object in mathematics.



Corn circle of the chaotic kind

The possibility that intelligent aliens have mastered fractal geometry just a couple of years after humans managed it seems unlikely. Instead suspicion has fallen on the University of Cambridge mathematics department. "We know nothing about it," claimed Peter Landeshoff. Benoit Mandelbrot, the IBM researcher who discovered the set, also denies any connection. He says he was in North America all last week.

The image went unrecognised by the *Cambridge Weekly News* which published this picture last Thursday. On Friday the paper printed a list of readers who pointed out that it probably owed more to Unemployed Fractal Observers than the other kind of UFOs. Unlike traditional shapes, fractals like the Mandelbrot set cannot be drawn with a pencil, gusts of wind or a scythe. They have to be built up dot by dot. However, mathematicians have recently proved that it is impossible to draw the Mandelbrot set accurately, even dot by dot. A really intelligent alien would have known that.

Mandelbrot munchies

In regard to the Mandelbrot corn-circle, you correctly state that it is impossible to draw the Mandelbrot set accurately dot by dot, as the images would be constructed in a cornfield. This would suggest we look ever closer at the *implications* of cornfields.

Within the swirls and seahorses of the set, once iterated, small but perfect Mandelbrot sets present themselves. Where should we look for these subsets?

I am pleased to be able to enclose the first piece of sold evidence towards a proof of the hypothesis that a higher intelligence is attempting to establish fractal communication with us. You will note that the ridge near the point of the main set visible on the cornflake consists of a series of peaks that may well be worth further examination under a scanning electron microscope.

Mandelbrot pop

Regarding the Mandelbrot cornflake, it follows that if a Mandelbrot cornflake is the self-same at all scales, then the Universe originated not with a big bang but a snap, crackle and pop.

Reprinted from *New Scientist* (24 August 1991) p.16 ff.

How To Spin $\mathcal{M}$-sets On A Monitor Screen In Real Time

John MacManus

A while back I came across an ad for a program called AcroSpin[1] which promised to display 3D objects on a PC screen in any video mode. In addition, the program promised to permit the normal manipulations of scaling, translating, and rotating in real time. Believe it!

The input data file for AcroSpin is an ASCII list of XYZ coordinates, and simple line, point, and color statements. The input file can be written in any vanilla text editor. My first input file was written by hand in my word processor with instructions to join some 350 atoms of a protein molecule in 3-space. It was worth it. The result was a protein backbone rotating smoothly on my home AT. Now I use semiautomatic ways to translate protein coordinates from a protein data base into AcroSpin format.

However, my interest in graphics is also a hobby, and so blossomed an interest in adventures in the complex plane. I looked at all the pretty pictures in the shiny books and drooled. I worked my way through the code in Becker and Dörfler's book *Dynamical Systems and Fractals*[2] and grew my own fractals at home. I was particularly interested in getting as much information onto the screen as possible, and so was particularly attracted to the pseudo-3D plots produced by incrementally offsetting the *X*-dimension to the right. I assigned the iteration number to them third dimension, and plotted it in the *Y*-dimension on the screen. This is done very cleverly by Becker and Dörfler from front to back with hidden line removal. An alternate approach by Ken Phillip with similar results recently came to my notice in a back issue of AMY (#13, October 1988).

But why not *real* 3D images? Why not output $\mathcal{M}$-set coordinates to AcroSpin using the iteration number as the *Z* dimension? Then one could rotate the fractal to one's heart's content. AcroSpin has a limit of about 10,000 points according to the manual. Since an EGA screen has 224,000 points, a full screen image is out of the question, but how about limiting oneself to a quarter EGA screen (56,000 points), and plotting every other pixel in both directions? That would give a usable file of 14,000 points. However, the data file is over 100 Kbytes, and the images are very sluggish to move because of the large block of points inside the $\mathcal{M}$-set.

How do we lighten up the image? This can be accomplished by plotting only a few points on the base plane to indicate its horizontal dimension. In addition, once inside the $\mathcal{M}$-set, instead of drawing each point, lines may be drawn between X_{min} and X_{max} for any given Y. This reduces data files to 40 - 60KB, which will rotate beautifully. AcroSpin sucks the entire data file into memory, and automatically scales the final image to be nearly full screen. The image can be scaled down to fill in the holes used to decrease data size. It can be viewed in whatever video mode (monochrome or color) is available to the user, including CGA, EGA, or VGA.

Another interesting feature of AcroSpin is layering of the image. In this case, ranges of iteration number can be assigned not color only, but also different layers of the image. This permits dissection of the image's iteration profile, and permits analysis if only monochrome is available.

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[1] From Acrobits, PO Box 5563, Redwood City, CA 94063-0563; $30.00.

[2] Cambridge University Press, 1989)

The program to produce output in AcroSpin format was written in MicroSoft's QuickBASIC. The $\mathcal{M}$-set algorithm is the usual one familiar to Amy readers, and consists of a mapping technique along the *X*-axis with incremental *Y* including the common test for inside or outside the set. If the point is outside the $\mathcal{M}$-set, then the *Z* dimension is assigned the iteration number. The *X, Y,* and *Z* are written to disk with the AcroSpin point notation. If the point is inside the set, then that point is flagged in an array sized to the *X* dimension. Before incrementing the *Y* dimension, the array is reexamined for beginnings and ends of the set boundary. The boundary *X* points (for later line drawing), *Y* and *Z* (the latter being the inputted maximal iteration) values are plotted to disk. Alterations in coordinates, iteration number, color, or layer can be made at the user's discretion.

The QuickBASIC listing to produce output of $\mathcal{M}$-set coordinates suitable for display by AcroSpin can be obtained for postage cost from: John P. MacManus / 38 Fentiman Ave. / Ottawa, Ontario, K1S 0T6 / CANADA.

Effects Of The Shape Of The Escape Region On $\mathcal{M}$-set Images

(Hey Charley—where's the T-shirt?)

Ken Philip

In AMYGDALA #9, Charles Fitch presented an image of a region in west Sea Horse Valley that showed a very odd structure consisting of connected lines of black and white blobs that ran perpendicular to the normal contours seen in Mandelbrot Set images. There was also a color slide showing the same region, which showed a progression of color shades along the rows of blobs — suggesting that the connected black (or white) blobs did not have the same dwell. He then offered to provide a T-shirt to the first person who could discover the boundary shape for the escape region used to create these images.

I had not made a determined effort to solve this puzzle until recently, when a question about the influence of the shape of the escape region on the resulting image turned up on the FracL-L conference on BITnet. That brought Fitch's puzzle to my attention again, and I began modifying a True BASIC Mandelbrot program in an attempt to reproduce his figure. I started out by looking into wrapping trig functions around the origin, letting the escape 'radius' be a sine or cosine function of the angle. I got some interesting-looking images, but nothing like Fitch's until I made a mistake and ended up with a large coefficient on a cosine function. Figure 5 shows the result, and indicates that I was on the track of the solution, if not very near it. Although the shapes of the black blobs were nothing like those in Fitch's image, the blobs were arranged along rows perpendicular to the dwell bands. I realized that the important thing here was that the value of *r* got quite large at certain angles — and looked for other functions that had the same property.

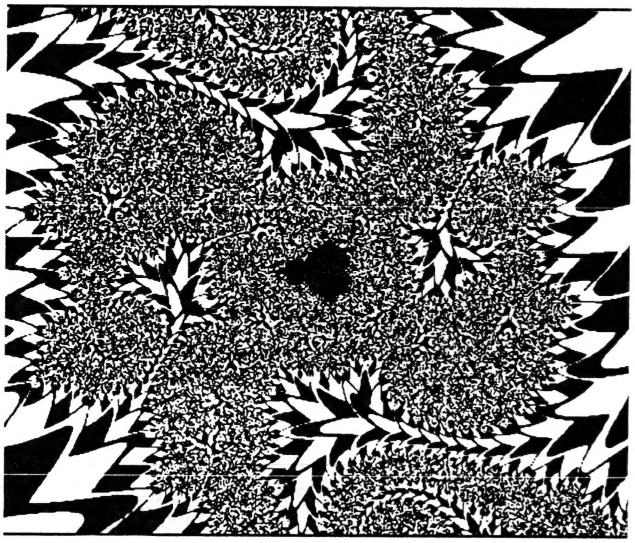


Figure 5. **Fitch's puzzle picture**
Alternate dwells black and white
Escape region $2 + 10\sqrt{1 - |\cos 2\vartheta|}$

Figure 6 (next page) shows the image for a function utilizing the tangent of the angle. The differences between this and Fitch's image are rather small.

I had been graphing the trig function in polar coordinates to see what the escape region looked like, and it became apparent that the best match to Fitch's image was produced by an escape region that was approaching two vertical lines symmetrically placed around the origin, as indicated in Fig-

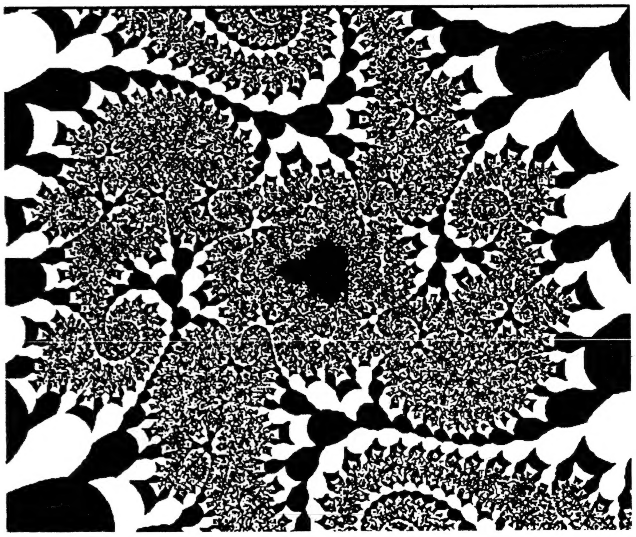


Figure 6. **Fitch's puzzle picture Alternate dwells black and white**

Escape region $3 + \sqrt{\tan\vartheta}$

Figure 7.

The dotted circles about the origin in Figure 7 are the 'normal' escape radius $r = 2$. At this point the light bulb finally turned on. Since I suspected that Fitch had used something simple, I tried testing for the real part of Z going less than -2 or greater than +2, while ignoring the imaginary part of Z (see the right hand image in Figure 7).

The result was Figure 8, which is a precise match of Fitch's figure in AMYGDALA #9.

Figure 9 shows a MandelZot image of the same area for comparison.

So the puzzle was solved, if by a rather round-about route...

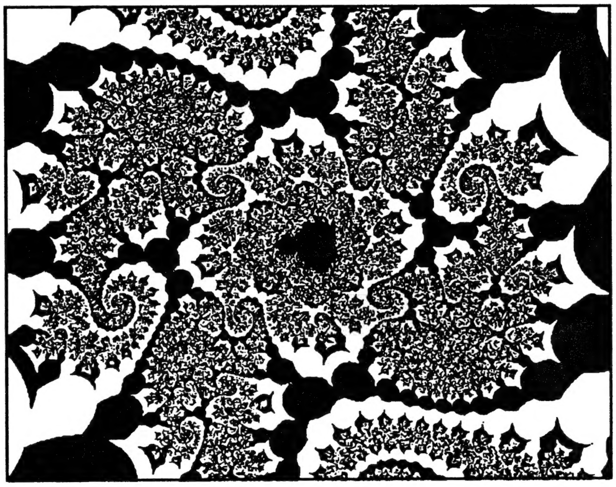


Figure 8. **Fitch's puzzle picture; True BASIC Mandelbrot program; Alternate dwells black and white; Escape region** $-2 < Re(c) < 2$

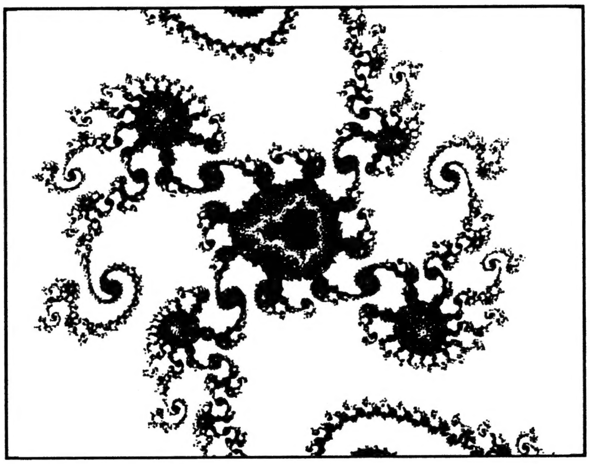
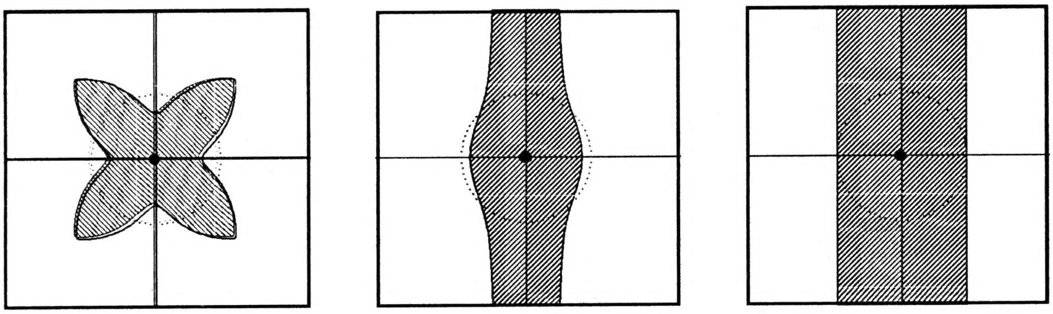


Figure 9. **Fitch's puzzle picture MandelZot image, dwell-band width = 100**



Escape region for Figure 5 Escape region for Figure 6 Escape region for Figure 8

Figure 7. **Escape Regions**

I should add that, as inspection of Figure 5, Figure 6 and Figure 7 will show, one can produce images of the $\mathcal{M}$-set when the escape radius is set to *less than* 2, and the images can be reasonably well-behaved provided the region of interest is itself well inside of the escape radius.

More On Fractal WitchCraft and Synchronous Orbits

Steve Stoft asked for space to reply to Leonard Herzmark's letter which appeared in Amy #26 as *A Review Of Fractal WitchCraft.* I'm reluctantly acceding to that request — "reluctantly" because I hesitate to perpetuate what may be moving towards an unseemly spat. "Acceding" because I guess I feel that Steve should have the right to reply.
Also, I apologize for leaving the title of Leonard's letter out of the Table of Contents in Amy #26.

Leonard Herzmark was twice asked to review FW (Fractal WitchCraft). On the first occasion he complained that none of our list of commonly used commands worked. After a phone call and several days of scratching my head, I finally deduced that Leonard had ignored the only instruction at the top of the list: "Use these from the Main Menu." He had just used them from wherever he happened be.

So we added a "/" command to FW that returns you from anywhere to the main menu, and began every command sequence in the list with a slash. This time Leonard had no problem.

Well, none till he tried entering coordinates: "of course not being able to enter the *Ymin,* I did not get the same graphics." Fortunately he sent me his four "NOT" maps with the wrong images. Looking at his entered coordinates I found what I suspected; Leonard had entered the *Ymin* value into the *Ymax* input slot. He had then wanted to enter *Ymax* into what he correctly saw to be the *Ymin* slot, but as FW tells you, it just calculates this value form the other three. Leonard had no problem with FW, he was just unable to figure out which of the two positive *Y* values given in B-of-F was the bigger number! Entering *Ymax* in the *Ymax* slot gives the right graphic every time.

If you have trouble telling which of two numbers is bigger you may still want to purchase FW for the trippy slide show. ☺

Bert Tyler, one of the co-authors of Fractint, I believe, sent Steve the following assessment of FW:

I'll confirm that FW's algorithm appears to be significantly faster than Fractint's when used with images that require high maxiter values. It would appear that you folks have found another way to beat brute force with intelligent algorithms. Congratulations!

Book Review

Review by Ian Entwistle

Fractal Geometry — Mathematical Foundations and Applications, Kenneth Falconer. Wiley (Chichester & New York, 1990), hard cover, 310 pp. $19.95

The contents page of the book lists eighteen chapters. Chapter headings range from *Hausdorff measure and dimension, Dynamical systems, Projection of fractals, Iteration of complex functions — Julia sets,* to *Random fractals* and *Multifractal measures.*

Most topics are treated by a section on background followed by theorems or propositions. Many examples follow. Each chapter is ended with a list of exercises.

Although the consequences of many of the proofs may be of interest to the general fractal enthusiast, this is a textbook for mathematicians or mathematics students.

Many of the diagrams, which include Koch curves, coastlines, Julia sets, and various attractors, will be familiar to most casual readers. The accompanying maths text will not be so easily followed.

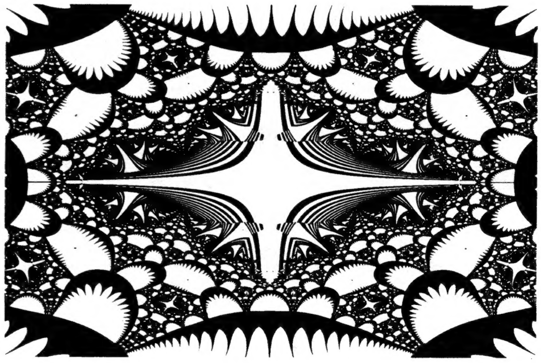
The substantial reference list (alphabetical by author) is up to date.

This book will convince most mathematicians unfamiliar with fractal geometry that fractals are not just the pretty pictures being generated by computers to please the media and pattern generators. They are seen to embody complex and fascinating mathematics. This book should be all a mathematician needs for conviction.

In the generally more readable introductory chapter, the author adopts a more rigid view about fractal than has developed in other recent texts. His detailed mathematical treatise therefore carefully avoids the wider application of fractals declared by B. Mandelbrot.

For Amygdala readers who are fractal generators the book cover illustration will be of interest. A nice coloured rendering of the Julia set for $z^2 + 0.233 + 0.5378i$ certainly attracts the eye.

This book is not recommended unless you are a trained mathematician.



Author

—Mike Freeman

Born and lived almost all my life in Vancouver, B.C. Degrees in Physics from UBC and Cal Tech. My primary occupation is teaching physics at Capilano College (2 yr.), but for the last 10+ years I've been doing programming and systems analysis for the college. My attraction to the Mandelbrot Set is twofold: the fascination with the set itself, and the challenge of writing better and faster programs to display it.

Books

— from the Springer Newsletter, Computer Science #1/92

Fractals for the Classroom, H.-O. Peitgen, H. Jürgens, D. Saupe, et. al.
Four new books on fractals and chaos. The authors, with help from leaders of the teaching community, have created books which are setting standards for science and teaching. The well written, clearly illustrated texts explore both the history and the unlimited potentials of fractals. Activities books encourage participation with exercises that are designed to develop a basic mathematical understanding and appreciation of fractals.

The first two volumes consist of independent chapters covering the major concepts related to the science and mathematics of fractals. Written at the mathematical level of an advanced secondary or beginning undergraduate student, *Fractals for the Classroom, Parts I & II* include many fascinating insights for the instructor, and integrate illustrations from a wide variety of applications with an enjoyable text to bring the concepts alive. These texts will have quite an impact upon professors, students, and the mathematics education of the general public.

The second two volumes discuss strategic activities which are designed to develop, through a hands-on approach, a basic mathematical understanding and appreciation of fractals. The exercises have been developed from a sound instructional base, stressing the connections to the contemporary curriculum as recommended in the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics. The slide package that accompanies the first volume includes some of the highest quality fractal images available anywhere.

Introduction to Fractals and Chaos; 450 pp., 289 illus., hardcover $29.00 (1992).

Complex Systems and the Mandelbrot Set; 384 pp., 141 illus., hardcover $29.00 (1992).

Strategic Activities, Volume I; 128 pp., 143 illus., 9 color slides/ Softcover $19.95 (1992).

Strategic Activities, Volume II; 187 pp., 75 illus./ Softcover $19.95 (1992).

The Science of Fractal Images, H.-O. Peitgen and D. Saupe.
"Anyone interested in the internal logic of the fractal world will find the book irresistible."
—*Nature*

Offering a comprehensive survey of the fractals that are most interesting from a graphics perspective, this "how-to" book demonstrates the way to create these striking images on a computer. Contributors include M.F. Barnsley, R.L Devaney, B.B. Mandelbrot, H.-O. Peitgen, D. Saupe, and R.F. Voss, with additional contributions by T. Fisher and M. McGuire.

312 pp., 142 illus., 39 color plates/ Hardcover $39.95 (1988).

Fractals and Chaos, T. Crilly, R.A. Earnshaw, and H. Jones.
What are the relationships between the two domains: chaos and fractals? This volume brings together a number of distinctive contributions concerning this central issue. The book provides an understanding of fractals, chaos, and the interrelationship between the two domains, leading to a common basis for examining the growth, development, organization, and behavior of complex dynamic systems.

277 pp., 146 illus./ Hardcover $39.50 (1991).

The Algorithmic Beauty of Plants, P. Prusinkiewicz and A. Lindenmayer.
This book explores mathematical models of the developmental processes and structures of plants, and illustrates them by using state-of-the-art computer-generated images. The authors outline the modeling and rendering of plants which are suitable for realistic image synthesis; the scientific potential of computer graphics in the visualization of biological structures and processes; the relationship between control mechanisms employed by living plants and the resulting complex developmental sequences and structures, and the relationship between developmental processes, self-similarity, and fractals.

228 pp., 150 illus., color plates/ Hardcover $39.95 (1990).

- The software and all the books are available from Amygdala (see Price List / Order Form).

Fading Away?

Well, we're down to 528 subscribers. Are people losing interest in fractals? Is it because of the long hiatus in publication over the past year? Has this newsletter become irrelevant or boring? Those questions could only be answered by taking a market survey, if at all, so I'll regard them as unanswerable, except by each of me/you for him/herself.

I'll assume that many of you are interested in keeping Amygdala going. For that to happen I'm going to need your help to boost circulation. I believe there are several thousand potential subscribers out there; the problem is, how to reach them. I'm not interested in "selling" them, just in getting them to know about Amygdala. Here's what you can do:

1. Get fractal friends to subscribe.

2. If you publish a book or periodical, think about mentioning Amygdala in it.

3. If you are active on electronic forums or bulletin boards where fractals are of interest, please mention Amygdala there.

4. If you read publications in which fractals are a topic, send a letter mentioning Amygdala.

5. If you're sharing your issues with someone else, get them to subscribe on their own. Saving money by piggybacking may be appropriate for big, rich publications like the Wall Street Journal, but it may just serve to kill the Amygdalan goose.

Circulation

As of 27 July 1992 Amygdala has 528 subscribers, 167 of whom have the supplemental color slide subscription.

Renewal

For 33 of you subscribers out there this is the last issue of your current subscription. You can determine if you are one of those by looking at the lower left of your mailing label: If it reads "A27" this is your last issue. if it reads "S27" this is your last slide set.
I urge you to use the enclosed form to renew your subscription promptly to avoid missing anything.

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July 27, 1992• These prices supersede those on all earlier price lists.

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Cellular Automatist Newsletter #1-10	$25.00 / *$32.00* / $39.00	☐

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Newsletter issues **#0a-25**	each $3.00 / *$3.20* / $3.90	☐

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"The Secant Method...(10) (Amy #25)", ten color slides from which the article's illustrations were taken	$20.00 / *$20.15* / $21.40	☐
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July 27, 1992